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## Review of PhD thesis in Theoretical Physics of Alexssandre de Oliveira Junior "Geometric and Information-Theoretic Aspects of Quantum Thermodynamics"

In this report I will provide a review of the thesis of Alexssandre de Oliveira Junior entitled "Geometric and Information-Theoretic Aspects of Quantum Thermodynamics". I will summarise the results obtained in each chapter, and make a final recommendation.

### Brief summary

As a brief summary, my view is that this is an excellent thesis, written to an extremely high standard, with a significant amount of care and attention, which has made it a pleasure to review. The thesis contains four results chapters, each detailing an independent and substantial contribution to the literature on quantum thermodynamics. The results contained within the first three such chapters have been published in peer-reviewed journals. The results in the final such chapter have appeared online as a pre-print, and I understand them to be currently under review.

### General remarks

Before going into further detail, there are two aspects regarding the candidate, his PhD studies, and the thesis, that I would already like to highlight at this stage. First, while this thesis contains results from four papers, the candidate in fact published another three papers (making seven in total) and has one other preprint (making two in total) during the course of his PhD studies. This is a huge achievement, and definitely singles the candidate out amongst his peers. Second, as alluded to already briefly above, this thesis is extremely well written, with a high-level of coherence between all the chapters, and a significant attention to detail. I would particularly commend the use of the 'tufte' or 'Feynman-lecture' style, involving a side-bar where many figures are placed, as well as the use of colours to code the different environments (e.g. differentiating theorems from corollaries and definitions).

### Subject of the thesis

The subject of this theoretical thesis is quantum thermodynamics. This is an active and important area of modern research which studies the physics at the intersection of two cornerstones of physics, namely quantum physics, and thermodynamics. It aims to apply and generalise the laws of thermodynamics, so that they apply for individual quantum systems (in particular, far from the traditional ‘thermodynamic limit’ of infinite system size), even when those systems are far from equilibrium, including coherent quantum phenomena. This field is a thriving field, and there has been a strong sub-community originating from quantum information science, that has been making significant developments over the past decade. This thesis is precisely in this direction, focusing mostly on the so-called ‘resource theory of quantum thermodynamics’, which is a powerful paradigm for studying the ultimate limits of thermodynamics in the quantum regime.

## Introductory and background chapters

The first two chapters of the thesis cover a very brief introduction to thermodynamics (from a modern perspective) and a summary of the results contained within the thesis respectively. The next two chapters contain the relevant theoretical background to the later results chapters. First, in chapter 3, the candidate provides an introduction to quantum theory, and introduces all the necessary background concerning the theory of majorization – a partial order on the set of probability distributions (relevant in the context of this thesis in particular for comparing incoherent – or diagonal – density operators). The candidate introduces not only the standard majorization, which is likely familiar to many readers, but also more specialised subjects, such as thermomajorisation, continuous thermomajorisation, approximate thermomajorisation and catalytic thermomajorisation. In Chapter 4, the candidate then goes on to give an introduction the resource theory of thermodynamics (mentioned above), as well as the more traditional open-system approach to quantum thermodynamics, based upon a continuous time Master equation. Both of these background chapters are well written, and provide a good level of rigour and detail, setting the stage perfectly for the results chapters to follow.

## Results chapter 1: thermal cones

The first results chapter is Chapter 5, entitled “Geometric structure of thermal cones”. This chapter presents results that were published by the candidate in first-author paper of the candidate with the same title, in the journal Phys. Rev. E.

This chapter is concerned with the fundamental questions of, for a given quantum state of a system (i) which states could the system have had in the past and (ii) which states could it evolve into in the future? In analogy to special relativity, the set of all such states a system could have evolved from is termed the ‘past thermal cone’, and the set of state it could evolve into the ‘future thermal cone’. All other states in the state-space are incomparable (being neither reachable from the current state, nor able to reach the current state). This structure can thus be seen to encode the thermodynamic arrow of time.

The chapter starts by considering the simplest case of either infinite-temperature or fully-degenerate energy spectrum, whereby transformation between states is governed by (standard) majorization. This allows the notion of a ‘majorisation cone’ to be introduced. The first main result is a complete characterisation of the past and future cones in this setting (Corollary 5.1.1 and Theorem 5.1.3), as well as the incomparable region (Lemma 5.1.2), thus giving a complete characterisation. Since majorization also occurs in the theory of pure-state bipartite entanglement, these results also immediately lead to a complete characterisation of the past and future cones of entanglement too.

Next, the results are extended to the general setting of thermodynamics, where analogous results are also obtained, leading to a complete characterisation of the past and future thermal cones (Theorem 5.2.6 and Corollary 5.2.3 respectively), as well as the incomparable region (Lemma 5.2.5). In order to obtain these more general results, the candidate had to introduce a novel structure (which may well be of independent interest in future research), which they termed a (temperature-dependent) "embedding lattice", which comes with a sufficient amount of structure that it can handle the subtleties of thermomajorisation over (standard) majorisation.

Finally, this chapter also studies the relative volume of the past and future thermal cones. The candidate shows that the relative volume are so-called 'thermodynamic monotones' – meaning that it is necessary that the relative volume decreases in a thermodynamic process (which can be viewed as a type of second-law-like inequality).

This is a comprehensive chapter, which contains a number of important and interesting results. The results are rather general (considering, as does much of the literature, energy incoherent states), and I believe it will be a significant challenge to generalise them further (e.g. to energy coherent states).

### Results chapter 2: memory-assisted Markovian thermal processes

The second results chapter is Chapter 6: "Memory-assisted Markovian thermal processes". This chapter presents results that were published by the candidate in a paper entitled "Thermal Recall: Memory-Assisted Markovian Thermal Processes" in the journal PRX Quantum.

The central question addressed in this chapter is how to quantify the role of memory in quantum thermodynamics. The resource-theory approach to quantum thermodynamics is typically interested in the ultimate limits of thermodynamics, and as such, it does not distinguish or quantify directly when a process needs to make use of a memory. This can be considered as a drawback to this paradigm, which should be filled, and it should be noted that it is in stark contrast to other approaches to quantum thermodynamics, in particular the Markovian – master equation – approach, which places large limitations on the behaviour of the system, forcing it to be both continuous-time, and memoryless. A key question is how to build a bridge between these two paradigms, and in the process, how to quantify the role of a memory in quantum thermodynamics?

This chapter announces results in this direction. In particular, it introduces a novel framework, termed "memory-assisted Markovian thermal processes", which provides a clear method for interpolating between the fully-Markovian regime, and the resource-theory regime. To do so, it uses recent results characterising Markovian processes in terms of continuous thermomajorisation. The key idea is to then explicitly include a finite sized memory – an arbitrary system at thermal equilibrium – in the process. This effectively singles out a small (finite dimensional) portion of the bath over which we have significantly more control (as in the resource-theory perspective), compared to the remainder of the bath, over which we assume we only have very limited (continuous-time and memoryless) control.

Starting in the infinite temperature limit, the first main result presented (Theorem 6.2.2) shows that an arbitrary state in the future thermal cone can be (approximately) reached, using a memory-assisted Markovian thermal process, with the error decreasing with the size of the memory.

Beyond the infinite-temperature limit, this chapter also announces two further results: In Theorem 6.2.4, for an interesting subset of states in the future thermal cone (those related to the current state by so-called 'beta-swaps', an elementary type of thermal interaction), it is shown that again in the limit of large memory, that every such state be approximately reached. Such beta-swaps in principle can be composed to reach *any* extremal state in the future thermal cone, however, the approximation would break down after a sufficiently big number of repetitions, and hence this result seems to imply, conversely, that not all extreme states in the future cone can in fact be reached.

As application of these results, this chapter then shows how to quantify the role of memory is a number of important thermodynamic tasks, including work extraction and cooling of two-dimension (qubit) systems.

An important aspect of the methodology presented in this work is that it is *model independent* – not relying on any specific model for Markovian dynamics, but in fact giving a model-independent bound, based solely on the size of the memory, which should makes the results particularly useful and widespread. As in the previous chapter, the results here concern only energy-incoherent states, however I believe it would again be very challenging to generalise them to the more general setting.

### Results chapter 3: Fluctuation-dissipation relations

The third results chapter is Chapter 7: Fluctuation-dissipation relations for thermodynamic distillation processes". This chapter presents results of a paper published by the author with the same title, in the journal Phys. Rev. E.

A central, powerful result of (statistical) thermodynamics is the so-called fluctuation-dissipation relation (or theorem), which shows (roughly speaking) that there is a fundamental link between the amount of dissipation that takes place in a process, and the existence of thermal fluctuations (in related processes). Given that the resource theory of quantum thermodynamic aims to reproduce and generalise all of the results of statistical thermodynamics (or to understand when potentially this isn't the case), it is an important question to ask what precise form such fluctuation-dissipation relations take in this setting?

This chapter outlines a fascinating approach to finding what can be viewed as quantum-thermodynamic fluctuation dissipation relations, in a setting with only a finite (but large) number of independent – but not necessarily identical – quantum systems. There are four main theorems presented in this direction. The first two relate to incoherent quantum systems: Theorem 7.3.1 establishes the optimal error that must be incurred when transforming one system into another, which depends only upon the ratio of the change in free energy of the system and the fluctuation in the free energy in the asymptotic limit, with a correction for finite N that also depends upon essentially the third moment of the distribution. In Theorem 7.3.2 it is furthermore establishes that when focusing on such an optimal protocol, the amount of free energy that is dissipated in the process is in fact proportional to the fluctuations in the free energy of the system.

The second pair of results presented are similar in nature, but now consider instead that the initial state of the process is a *pure quantum state*. The two results (given in Theorems 7.3.3 and 7.3.4) are largely analogous to the previous two theorems, except now a natural restriction on the Hamiltonian of the system needs to be in place (namely, that it has no degenerate energy gaps, which is a common assumption necessary in results of this type found in the literature).

The chapter also outlines a number of interests consequences of this general fluctuation-dissipation theorem. In particular, it can be used to further analysis optimal work extraction protocols, where it can be used to derive so-called ‘second-order asymptotic corrections’ (i.e. corrections to the standard thermodynamic limit, which hold in the regime of large-but-finite system sizes). What is particularly nice is that this correction is now precisely seen to be governed by the free-energy fluctuations of the system. In a similar fashion, for the reverse process of erasure, a similar analysis shows that the correction to the thermodynamic limit also arises due to fluctuations in free energy. Finally, in a more novel direction, the idea of ‘thermodynamically-free communication’ is considered, and it is shown how the fluctuation-dissipation relation obtained can be used to study the communication rate of such a protocol (beyond the infinite-message limit).

### Results chapter 4: Quantum catalysis in cavity QED

The fourth and final results chapter is Chapter 8: Quantum catalysis in cavity quantum electrodynamics. This chapter presents the results from a first-author preprint of the candidate with the same title.

Quantum (thermodynamic) catalysis is a fascinating theoretical phenomenon, whereby an auxiliary system can be employed in a thermodynamic process – and returned unchanged at the end – to make possible transformations which are provably impossible without the additional system present. There has been a long history of studying quantum catalysis, it remained an open question as to whether this is practically relevant, or merely a theoretical possibility.

This exciting chapter demonstrates that quantum catalysis indeed has the potential to be relevant in the context of cavity QED. In particular, it is shown that in the paradigmatic Jaynes-Cummings model (modelling the interaction of an atom with the electromagnetic field inside the cavity) the atom can in fact act as a catalyst for preparing quantum states of the cavity.

More precisely, it is shown that the second-order coherence of the final state of the cavity witnesses that a non-classical state of the cavity is prepared (thus, in particular, demonstrating in a meaningful sense that this is a truly quantum process), given that the initial state is the (classical) coherent state.

Going beyond this mere existence result, this chapter explores the question of how general quantum catalysis might be in cavity QED more generally. Surprisingly, the investigation of the candidate shows that in this setting catalysis appears to be rather generic, which is an exciting finding. It is notably that it is in the regime of low power where catalytic effects appear to be most pronounced, and this could have interesting applications.

## Summary

In summary, this thesis makes a significant contribution to the field of quantum thermodynamics, announcing a range of results from strengthening the extending the resource theory of quantum thermodynamics, as well as demonstrating potential applications within cutting-edge physical platforms. This thesis is particularly timely, as the field of quantum thermodynamics continues to grow, and its connections and applications in near-term quantum technologies becomes ever more relevant. I expect that numerous results from this thesis will have a long-term impact on the field, and will lead to significant further exploration. For these reasons, as well as those outlined above, I make the following recommendation:

## Final recommendation

I, the undersigned, hereby certify that the reviewed doctoral dissertation of Mr. Alexssandre de Oliveira Junior meets the statutory requirements set in the Polish Act - Law on Higher Education and Science for doctoral theses in the fields of natural and exact sciences within the discipline of physical sciences. I request its acceptance by the Discipline Council for Physical Sciences at Jagiellonian University and its authorization for public defence.

Furthermore, given the overall excellent quality of the thesis, both in terms of the results it contains, but also in terms of the care and attention that was taken in presenting these results in a coherent and complete manner, I definitely recommend to award the thesis with summa cum laude distinction.

Yours Sincerely

P Skrzypczyk

Dr Paul Skrzypczyk