Cyclic vectors in Dirichlet-type spaces in the unit ball of $\mathbb{C}^n$

Abstract:

The present doctoral thesis studies mainly the cyclicity problem of polynomials in Dirichlet-type spaces in the unit ball of $\mathbb{C}^n$. The classical Hilbert spaces of holomorphic functions are the Hardy space, the Bergman space and the Dirichlet space. All of them are special cases of a space family that depends on a real parameter, called Dirichlet-type spaces. Studying this family one may draw conclusions for the above well known spaces that they play a key role in a couple of research fields in mathematical analysis.

Many researchers working in Dirichlet-type spaces and their fruitful theory are interested in the problem of characterizing cyclic vectors, yet at the same time they make an effort to answer questions coming up in the investigation. For instance, some of them are: other characterizations of these spaces, equivalent integral norms, norm comparisons, sharp decay of norms, shift-invariant subspaces, reproducing kernels, compositions operators, multipliers, zero sets, capacity conditions of non-cyclicity e.t.c..

Our aim is to characterize the polynomials that are cyclic with respect to the shift operators in Dirichlet-type spaces in the unit ball of $\mathbb{C}^n$, which involves identifying whether a polynomial is cyclic at a fixed parameter.

Keywords: Dirichlet-type spaces, cyclic vectors, polynomials in several variables