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Relation on the Thesis:

Non-product geometries for particle physics and cosmology

by *Arkadiusz Jakub Bochniak*

There is a belief in part of the theoretical physics community that the unification of gravity and field theory must pass trough a rethinking of the very structure of spacetime at short (Planckian) distances. A similar phenomenon happened with classical particles and their phase space. The birth of quantum mechanics meant that the phase space had to be described not by a standard manifold, but by a "pointless geometry" described by noncommuting operators acting on a Hilbert space. Quantum phase space is the first example of *noncommutative geometry.*

In order to study such generalised space one has first to generalise the very concept of geometry. This is what is done in the first part of the thesis, which ascts as a prelude to the original part in chapter two.

The generalisations goes from the study of points to that of algebra, or the fields defined on the space. Different kind of algebras capture different characteristics. In particular C^*-algebras connect to topology, other $*$-algebras to differentiability. Indeed the thesis describes the algebrisation of differential calculus and of differential forms in this context. The $*$-algebras are represented as operators algebra on some Hilbert space. The main actors are defined inthis chapter, like Fredhom modules, Hochschild cohomology, gradings.

All this culminates to the construction of spectral triples formed by an algebra represented on an Hilbert space, and a generalised DIrac operator which gives metric, differential calculus and other geometrical aspects. The ordinary spacetime manifold can be described by such a spectral triple, with a commutative algebra. The thesis generalises to noncommutative algebras. Other structures and conditions are introduced. Another actor which will be important in the following and which appears in the first chapter is the twist.

The thesis has a strong emphasis on the differential calculus is a trademark of the Krakow school on noncommutative geometry.

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Things come together with the construction of the spectral action. This is presented in the usual Chamseddine and Connes approach, mainly useful for field theory, but also as a Wodzicki residue, useful to see the gravitational point of view.

All this will be applied to the standard model of fundamental interactions in the first part of chapter two, which as I said is original. The fermionic and bosonic action are introduced. There are some comments on the phenomenology, which could have been expanded. There are highlight fermion doubling, which bridge to a Lorentzian presentation. So far in fact everything has been Euclidean. The connection between these two aspects, noted not long ago, are key to some of the original contribution, which start in the second chapter.

Until recently far all studies where in the context of product geometries, also called almost commutative geometries, some sort of Kaluza-Klein. The paper *Spectral geometry for the standard model without fermion doubling* in PHYSICAL REVIEW D 101, 075038 (2020) introduces in this context the possibility of nonproduct geometries. An important contribution.

The fundamental idea, which I find very astute, is not to change of the algebra, but the way it acts on the Hilbert space, and how it intertwine with the Lorentz signature issue. This is solved generalising the Hilbert space to a Krein space. Likewise the generalised Dirac operator is 'shifted' to a Krein version.

The paper while not being the last word (it has some drawbacks, including CP violations) is a decisive step forward, and is likely that the ideas, once they have been metabolised by the community, will bring some concrete result, also form the phenomenological side of the problem. The issue is in fact further developed in the second paper presented in the thesis, *Spectral action and the electroweak θ-terms for the Standard Model without fermion doubling* JHEP12(2021)142, with most interesting applications to the electroweak sector of the standard model.

While the first part of the thesis (apart from the introduction) was more geared towards particle physics, the second is more related to gravity, and spacetime issues. In particular the applications of the nonproduct geometries to modified gravity and bimetric models. These are model for which two different geometries (metrics) co-exist on the same space. This is seen as a diagonal Dirac operator with different components acting on a doubled spaces. A generalisation of the original Connes-Lott model, in which however the two sheets have independent geometries. The first question to ask is the stability of the solutions. The question has for the moment an interesting partial answer, in that some stable solutions are found, as cases of the usual known solutions. This leads to the paper *Stability of Friedmann-Lematre-Robertson-Walker solutions in doubled geometries* Phys. Rev. D 103, 044041 (2021).

This result is generalised in a recent preprint *Spectral interaction between universes* arXiv: 2201.03839 where the two sheets are made to interact via non diagonal Dirac operators, just as it happens for the construction of the Higgs field in the Connes-Lott model. The interesting aspect is the comparison with the "simple" bimetric models. In this case the structure has an added richness worth receiving more studies. I add that this paper seems to be rather calculationally intensive, but the candidate and his collaborator managed to perform the calculations using interesting techniques.

A further generalisation, by the candidate as single author is in the preprint *Towards modified bimetric theories within non-product spectral geometry*, arXiv: 2202.03765. While so far the models had been of the Friedman-Robertson-Walker-Lemaitre type, in this case the metrics of the doubled geometries, as encoded by theDirac operator, is more general, the Hopf model. Probably these models are more apt for a numerical study, but a preliminary investigation, such as the one performed in this paper, is imprescindible.

To conclude, I found the thesis well written and the results of utmost interest, and I am favourable to the public defence of Arkadiusz Jakub Bochniak thesis, and I propose it to be approved with distinction.

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